Name: ___________________________

Math 12Honors (2023) Challenging Trigonometric Identities Problems: Solving and Proving (Part 1)

1. Prove the following:

   a) $\tan\left(45^\circ + x\right) - \tan\left(45^\circ - x\right) \equiv 2\tan 2x$

   b) $\dfrac{\cos x - 5\cos 5x + \cos 9x}{\sin x - 5\sin 5x + \sin 9x} = \cot 5x$

   c) $\tan\left(A+B\right) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

   d) $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

   e) $\cot\left(x+y\right) = \dfrac{\cot x \cot y - 1}{\cot x + \cot y}$

   f) $\cot\theta + \csc\theta \equiv \cot\dfrac{\theta}{2}$

   g) $\cot^2\theta + 1 \equiv \csc^2\theta$

   h) $\sin^2 A - \sin^2 B \equiv \sin\left(A+B\right)\sin\left(A-B\right)$

2. Using $\cos 3x = \cos\left(x + 2x\right)$, prove the following: $\cos 3x = 4\cos^3 x - 3\cos x$

3. Show that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

   b. Use the equation above to prove that $\sin 18^\circ = \dfrac{\sqrt{5}-1}{4}$

4. Show that if $A+B+C=\pi$ where $0<A,B,C<\pi$, then
$\cot A\cot B+\cot B\cot C+\cot C\cot A=1$

5. Show that if $w+x+y+x=\pi$ and where $0<w,x,y,z<\pi$ then:
$\cot w\cot x\cot y+\cot w\cot x\cot z+\cot x\cot y\cot z+\cot w\cot y\cot z=\cot w+\cot x+\cot y+\cot z$

6. Show that if $A+B+C=\pi$, then
$\sin A\sin B\cos C+\sin A\cos B\sin C+\cos A\sin B\sin C=1+\cos A\cos B\cos C$

7. Find the general solution of the equation:

a) $\sin 9x\sin 8x-\sin 7x\sin 6x=0$

b) $\sin\left(5x+\frac{\pi}{4}\right)=\sin 2x$

c) $\tan 2x\tan x=1$

d) $\cos x-\cos 3x\equiv\sin 2x$

e) $\tan 5\theta+\cot 2\theta=0$

8. Evaluate $\cos x$ if $\cos 3x=1$

9. Prove the following: $2\sin 40^\circ \sin 80^\circ = \cos 40^\circ + \frac{1}{2}$

10. Evaluate the following without using a calculator: $\sin 20^\circ \sin 40^\circ \sin 80^\circ = ??$

11. Given that $y = a\cos\theta + b\sin\theta$, where "a' and "b" are real values, express the function "y" in the form of $y = k\sin(\theta + \alpha)$, where $k > 0$ and $\frac{-\pi}{2} < \alpha < \frac{\pi}{2}$

12. Find the maximum and minimum value of $\cos\theta - 7\sin\theta$

13. Find the general solution to the equation: $\cos\theta - 7\sin\theta = 5$

14. If the maximum value of $a\cos\theta + b\sin\theta$ is equal to "M", show that the maximum value of $a\cos\theta - b\sin\theta$ is also "M".
    Show that in ΔABC, if $a^2 - b^2 = b^2 - c^2$, then $\sin(A-B)\sin C = \sin A \sin(B-C)$

15. Using the equation from the previous question, show that $\frac{1}{\tan A} - \frac{1}{\tan B} = \frac{1}{\tan B} - \frac{1}{\tan C}$

16. Express $\sin x$, $\cos x$, and $\tan x$ in terms of $\tan\frac{x}{2}$ [yes, sinx is the function and cosx is the function]

17. Given that $\sec x - \tan x = y$, use the equations from above to prove: $\tan\frac{x}{2} = \frac{1-y}{1+y}$

Using the sum and difference identities: $\sin(a+b) = \sin a \cos b + \sin b \cos a$ and $\sin(a-b) = \sin a \cos b - \sin b \cos a$, prove the following two equations:

a) $\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

b) $\sin x - \sin y = 2\sin\left(\frac{x-y}{2}\right)\cos\left(\frac{x+y}{2}\right)$

c) Prove that if $\frac{a}{p} = \frac{b}{q}$ then show that: $\frac{a+b}{a-b} = \frac{p+q}{p-q}$

d) Prove that in ΔABC, we have: $\frac{a+b}{a-b} = \frac{\tan\left(\frac{A+B}{2}\right)}{\tan\left(\frac{A-B}{2}\right)}$

18. Convert the equation: $\tan\theta - \sec\theta + 1 = 0$ into the form $a\cos\theta + b\sin\theta + c = 0$, where $a > 0$.

19. Given that $\frac{\sin^2 A}{1+3\cos^2 A} = \frac{3}{16}$, where $90° < A < 180°$, find the value of $\frac{\sin A}{1+3\cos A}$